In **Figure 5-2** the gear train has a difference of numbers of teeth of only 1; $z_1 = 30$ and $z_2 = 31$. This results in a reduction ratio of 1/30.

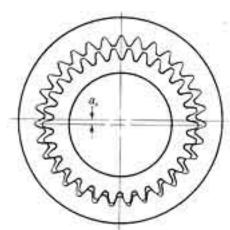


Fig. 5-2 The Meshing of Internal Gear and External Gear in which the Numbers of Teeth Difference is 1

 $(z_2 - z_1 = 1)$

SECTION 6 HELICAL GEARS

The helical gear differs from the spur gear in that its teeth are twisted along a helical path in the axial direction. It resembles the spur gear in the plane of rotation, but in the axial direction it is as if there were a series of staggered spur gears. See **Figure 6-1**. This design brings forth a number of different features relative to the spur gear, two of the most important being as follows:



Fig. 6-1 Helical Gear

- 1. Tooth strength is improved because of the elongated helical wraparound tooth base support.
- Contact ratio is increased due to the axial tooth overlap. Helical gears thus tend to have greater load carrying capacity than spur gears of the same size. Spur gears, on the other hand, have a somewhat higher efficiency.

Helical gears are used in two forms:

- 1. Parallel shaft applications, which is the largest usage.
- Crossed-helicals (also called spiral or screw gears) for connecting skew shafts, usually at right angles.

6.1 Generation Of The Helical Tooth

The helical tooth form is involute in the plane of rotation and can be developed in a manner similar to that of the spur gear. However, unlike the spur gear which can be viewed essentially as two dimensional, the helical gear must be portrayed in three dimensions to show changing axial features.

Referring to Figure 6-2, there is a base cylinder from which a

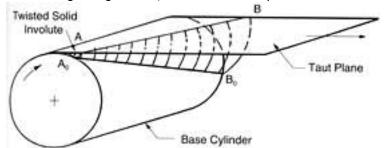


Fig. 6-2 Generation of the Helical Tooth Profile

taut plane is unwrapped, analogous to the unwinding taut string of the spur gear in **Figure 2-2**. On the plane there is a straight line AB, which when wrapped on the base cylinder has a helical trace A_0B_0 . As the taut plane is unwrapped, any point on the line AB can be visualized as tracing an involute from the base cylinder. Thus, there is an infinite series of involutes generated by line AB, all alike, but displaced in phase along a helix on the base cylinder.

Again, a concept analogous to the spur gear tooth development is to imagine the taut plane being wound from one base cylinder on to another as the base cylinders rotate in opposite directions. The result is the generation of a pair of conjugate helical involutes. If a reverse direction of rotation is assumed and a second tangent plane is arranged so that it crosses the first, a complete involute helicoid tooth is formed.

6.2 Fundamentals Of Helical Teeth

In the plane of rotation, the helical gear tooth is involute and all of the relationships governing spur gears apply to the helical. However, the axial twist of the teeth introduces a helix angle. Since the helix angle varies from the base of the tooth to the outside radius, the helix angle β is defined as the angle between the tangent to the helicoidal tooth at the intersection of the pitch cylinder and the tooth profile, and an element of the pitch cylinder. See **Figure 6-3.**

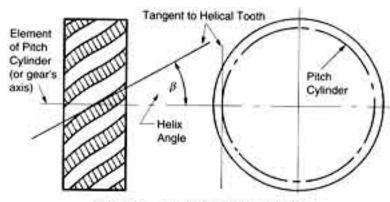


Fig. 6-3 Definition of Helix Angle

The direction of the helical twist is designated as either left or right. The direction is defined by the right-hand rule.

For helical gears, there are two related pitches - one in the plane of rotation and the other in a plane normal to the tooth. In addition, there is an axial pitch.

Referring to **Figure 6-4**, the two circular pitches are defined and related as follows:

 $p_n = p_t cos \beta = normal circular pitch$

The normal circular pitch is less than the transverse radial pitch, pt in the plane of rotation; the ratio between the two being equal to the cosine of the helix angle.

Consistent with this, the normal module is less than the transverse (radial) module.

The axial pitch of a helical gear, p_X , is the distance between corresponding points of adjacent teeth measured parallel to the gear's axis - see **Figure 6-5**.

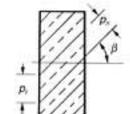


Fig. 6-4 Relationship of Circular Pitches

р,

Fig. 6-5 Axial Pitch of a Helical Gear

Axial pitch is related to circular pitch by the expressions:

$$p_X = p_t \cot \beta = P_{n-} = \text{axial pitch}$$
 (6-2)

A helical gear such as shown in Figure 6-6 is a cylindrical gear in which the teeth flank are helicoid. The helix angle in standard pitch circle cylinder is β . and the displacement of one rotation is the lead, L.

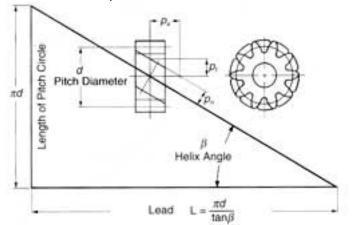


Fig. 6-6 Fundamental Relationship of a Helical Gear (Right-Hand)

axial view, or in the plane perpendicular to the axis, the helical gear has two kinds of tooth profiles - one is based on a normal system, the other is based on an axial system.

circular pitch, P_n And p_n , divided by p is then a normal module, mt

$$m_{n} = \underline{P_{n}}$$
 (6-3)

p
The tooth profile of a helical gear with applied normal module, mn, and normal pressure angle an belongs to a normal system.

In the axial view, the circular pitch on the standard pitch circle is called the radial circular pitch, Pt* And pt, divided by p is the radial module, mt.

$$m_{t} = \underline{P_{t}}_{p} \tag{6-4}$$

6.3 Equivalent Spur Gear

The true involute pitch and involute geometry of a helical gear follows is in the plane of rotation. However, in the normal plane, looking at one tooth, there is a resemblance to an involute tooth of a pitch corresponding to the normal pitch. However, the shape of the tooth corresponds to a spur gear of a larger number of teeth, the exact value depending on the magnitude of the helix angle.

The geometric basis of deriving the number of teeth in this equivalent tooth form spur gear is given in Figure 6-7. The result of the transposed geometry is an equivalent number of teeth, given as:

$$z_V = \underline{z}$$

$$\cos^3 b$$
(6-5)

This equivalent number is also called a virtual number because this spur gear is imaginary. The value of this number is used in determining helical tooth strength.

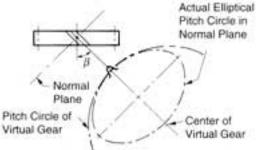
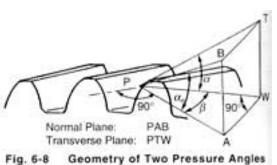


Fig. 6-7 Geometry of Helical Gear's Virtual Number of Teeth

6.4 Helical Gear Pressure Angle

Although, strictly speaking, pressure angle exists only for a gear pair, a nominal pressure angle can be considered for an individual gear.

helical gear there is a normal pressure, α_n , angle as well as the usual pressure angle in the plane of rotation, α . Figure 6-8 shows their relationship, which is expressed



(6-6)

 $tan\alpha = \underline{tan}\alpha_{n}$ cosb

6.5 Importance Of Normal Plane Geometry

Because of the nature of tooth generation with a rack-type The tooth profile of a helical gear is an involute curve from an hob, a single tool can generate helical gears at all helix angles as well as spur gears. However, this means the normal pitch is the common denominator, and usually is taken as a standard value. Since the true involute features are in the transverse Circular pitch measured perpendicular to teeth is called normal plane, they will differ from the standard normal values. Hence, there is a real need for relating parameters in the two reference

6.6 Helical Tooth Proportions

hese follow the same standards as those for spur gears. Addendum, dedendum, whole depth and clearance are the same regardless of whether measured in the plane of rotation or the normal plane. Pressure angle and pitch are usually specified as standard values in the normal plane, but there are times when they are specified as standard in the transverse plane.

6.7 Parallel Shaft Helical Gear Meshes

Fundamental information for the design of gear meshes is as

Helix angle - Both gears of a meshed pair must have the same helix angle. However, the helix direction must be opposite;

e., a left-hand mates with a right-hand helix. **Pitch diameter** - This is given by the same expression as for spur gears, but if the normal module Is involved it is a function of the helix angle. The expressions are:

$$d = z m_t = \underline{z}$$

$$m_n \cos \beta$$
(6-7)

Center distance - Utilizing Equation (6-7), the center distance of a helical gear mesh is:

$$a = Z_1 + Z_2$$
 (6-8)
 $2m_n \cos \beta$

Note that for standard parameters in the normal plane, the center distance will not be a standard value compared to standard spur gears. Further, by manipulating the helix angle, β , the center distance can be adjusted over a wide range of values. Conversely, it is possible:

- 1. to compensate for significant center distance changes (or errors) without changing the speed ratio between parallel geared shafts: and
- 2. to alter the speed ratio between parallel geared shafts without changing the center distance, by manipulating the helix angle along with the numbers of teeth.

6.8 Helical Gear Contact Ratio

The contact ratio of helical gears is enhanced by the axial overlap of the teeth. Thus, the contact ratio is the sum of the transverse contact ratio, calculated in the same manner as for spur gears, and a term involving the axial pitch.

$$(\varepsilon)_{\text{total}} = (\varepsilon)_{\text{trans}} + (\varepsilon)_{\text{axial}}$$
or
$$\varepsilon_{t} = \varepsilon_{\alpha} + \varepsilon_{\beta}$$
(6-9)

Details of contact ratio of helical gearing are given later in a general coverage of the subject; see **SECTION 11.1**.

6.9 Design Considerations

6.9.1 Involute Interference

Helical gears cut with standard normal pressure angles can have considerably higher pressure angles in the plane of rotation - see Equation (6-6) - depending on the helix angle. Therefore, the minimum number of teeth without undercutting can be significantly reduced, and helical gears having very low numbers of teeth without undercutting are feasible.

6.9.2 Normal vs. Radial Module (Pitch)

In the normal system, helical gears can be cut by the same gear hob if module m_n and pressure angle α_n are constant, no matter what the value of helix angle β

It is not that simple in the radial system. The gear hob design must be altered in accordance with the changing of helix angle β even when the module m, and the pressure angle $\alpha_{\text{t}},$ are the same.

Obviously, the manufacturing of helical gears is easier with the normal system than with the radial system in the plane perpendicular to the axis.

6.10 Helical Gear Calculations 6.10.1 Normal System Helical Gear

In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter d_W and working pressure angle α_{Wt} in the axial system is done per **Equations** (6-10). That is because meshing of the helical gears in the axial direction is just like spur gears and the calculation is similar.

$$d_{w1} = 2a_x \frac{Z_1}{Z_1 + Z_2}$$

$$d_{w2} = 2a_x \frac{Z_2}{Z_1 + Z_2}$$

$$\alpha_{w2} = \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a_x}\right)$$
(6-10)

Table 6-1 shows the calculation of profile shifted helical gears in the normal system. If normal coefficients of profile shift x_{n1} x_{n2} are zero, they become standard gears.

If center distance, ax is given, the normal coefficient of profile shift x_{n1} and x_{n2} can be calculated from **Table 6-2**. These are the inverse equations from items 4 to 10 of **Table 6-1**.

Table 6-1 The Calculation of a Profile Shifted Helical Gear in the Normal System (1)

No.	Item	Symbol	Formula	Example		
NO.				Pinion	Gear	
1	Normal Module	m _n		3		
2	Normal Pressure Angle	α_o		20°		
3	Helix Angle	β		30°		
4	Number of Teeth & Helical Hand	Z1, Z2		12 (L)	60 (R)	
5	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	22.79588°		
6	Normal Coefficient of Profile Shift	X X 2		0.09809	0	
7	Involute Function $\alpha_{_{ m wf}}$	inv $\alpha_{\rm wt}$	$2\tan\alpha_n\left(\frac{X_{n1}+X_{n2}}{Z_1+Z_2}\right)+\mathrm{inv}\alpha_1$	0.023405		
8	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	23.1126°		
9	Center Distance Increment Factor	у	$\frac{Z_1 + Z_2}{2\cos\beta} \left(\frac{\cos\alpha_t}{\cos\alpha_{st}} - 1 \right)$	0.09744		
10	Center Distance	a,	$\left(\frac{z_1+z_2}{2\cos\beta}+y\right)m_n$	125.000		
11	Standard Pitch Diameter	d	zm _n cosβ	41.569	207.846	
12	Base Diameter	d_b	d cosα _t	38.322	191.611	
13	Working Pitch Diameter	d _w	$\frac{d_b}{\cos \alpha_{wl}}$	41.667 208.3		
14	Addendum	h _{at} h _{a2}	$(1 + y - x_{n2}) m_n$ $(1 + y - x_{n1}) m_n$	3.292	2.998	
15	Whole Depth	h	$[2.25 + y - (x_{n1} + x_{n2})]m_n$	6.748		
16	Outside Diameter	d _a	d + 2 ha	48.153	213.842	
17	Root Diameter	d,	d _a - 2 h	34.657	200.346	

Table 6-2 The Calculations of a Profile Shifted Helical Gear in the Normal System (2)

No.	Item	Symbol	Formula	Exam	ple
1	Center Distance	a _x		125	
2	Center Distance Increment Factor	у	$\frac{a_x}{m_n} = \frac{z_1 + z_2}{2\cos\beta}$	0.097447	
3	Radial Working Pressure Angle	α_{w}	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha_1}{(z_1+z_2)+2y\cos\beta}\right]$	23.1126°	
4	Sum of Coefficient of Profile Shift	$X_{n1} + X_{n2}$	$\frac{(z_1 + z_2)(\text{inv }\alpha_{wt} - \text{inv }\alpha_t)}{2\tan \alpha_n}$	0.09809	
5	Normal Coefficient of Profile Shift	Xn1 . Xn2		0.09809	0

The transformation from a normal system to a radial system is accomplished by the following equations:

$$m_{r} = \frac{m_{o}}{\cos \beta}$$

$$\alpha_{t} = \tan^{-1}\left(\frac{\tan \alpha_{n}}{\cos \beta}\right)$$
(6-11)

6.10.2 Radial System Helical Gear

Table 6-3 shows the calculation of profile shifted helical gears in a radial system. They become standard if $x_{11} = x_{12} = 0$.

Table 6-4 presents the inverse calculation of items 5 to 9 of **Table 6-3**.

The transformation from a radial to a normal system is described by the following equations:

$$x_n = \frac{x_r}{\cos \beta}$$

$$m_r = m_r \cos \beta$$

$$\alpha_n = \tan^{-1} (\tan \alpha_r \cos \beta)$$
(6-12)

6.10.3 Sunderland Double Helical Gear

A representative application of radial system is a double helical gear, or herringbone gear. made with the Sunderland machine. The radial pressure angle, α_t and helix angle, β , are specified as 20° and 22.5°, respectively The only differences from the radial system equations of **Table 6-3** are those for addendum and whole depth. **Table 6-5** presents equations for a Sunderland gear.

6.10.4 Helical Rack

Viewed in the normal direction, the meshing of a helical rack and gear is the same as a spur gear and rack. **Table 6-6** presents the calculation examples for a mated helical rack with normal module and normal pressure angle standard values. Similarily, **Table 6-7** presents examples for a helical rack in the radial system (i.e., perpendicular to gear axis).

Table 6-3 The Calculation of a Profile Shifted Helical Gear in the Radial System (1)

No.	Item	Symbol	Formula	Example	
NO.				Pinion	Gear
1	Radial Module	m_t		3	
2	Radial Pressure Angle	α_1		20°	
3	Helix Angle	β		30°	
4	Number of Teeth & Helical Hand	Z1, Z2		12 (L) 60 (F	
5	Radial Coefficient of Profile Shift	X11, X12		0.34462	0
6	Involute Function $\alpha_{_{\mathrm{inf}}}$	inv α _{wt}	$2\tan\alpha_{t}\left(\frac{X_{tt}+X_{t2}}{Z_{t}+Z_{2}}\right)+\operatorname{inv}\alpha_{t}$	0.0183886	
7	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	У	$\frac{Z_1 + Z_2}{2} \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right)$	0.33333	
9	Center Distance	a,	$\left(\frac{z_1+z_2}{2}+y\right)m_i$	109.0000	
10	Standard Pitch Diameter	d	zm _t	36.000	180.000
11	Base Diameter	d _b	$d\cos\alpha_t$	33.8289	169.1447
12	Working Pitch Diameter	d _w	$\frac{d_b}{\cos \alpha_{wt}}$	-36.3333	181.666
13	Addendum	h _{at} h _{a2}	$(1 + y - x_{12}) m_t$ $(1 + y - x_{11}) m_t$	4.000	2.966
14	Whole Depth	h	$[2.25 + y - (x_{t1} + x_{t2})]m_t$	6.7	16
15	Outside Diameter	d _a	d + 2 h _a	44.000	185.932
16	Root Diameter	d,	d _a - 2 h	30.568	172.500

Table 6-4 The Calculation of a Shifted Helical Gear in the Radial System (2)

No.	Item	Symbol	Formula		
1	Center Distance	a _x		109	
2	Center Distance Increment Factor	у	$\frac{a_x}{m_t} - \frac{z_1 + z_2}{2}$	0.33333	
3	Radial Working Pressure Angle	αwt	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha_t}{(z_1+z_2)+2y}\right]$	21.39752°	
4	Sum of Coefficient of Profile Shift	$X_{t1} + X_{t2}$	$\frac{(Z_t + Z_2)(\text{inv } \alpha_{wt} - \text{inv } \alpha_t)}{2 \tan \alpha_n}$	0.34462	
5	Normal Coefficient of Profile Shift	X11 , X12		0.34462	0

Table 6-5 The Calculation of a Double Helical Gear of SUNDERLAND Tooth Profile

No.	ltem .	Symbol	Formula	Example	
140.				Pinion	Gear
1	Radial Module	m,		3	
2	Radial Pressure Angle	α,		20°	
3	Helix Angle	β		22.5°	
4	Number of Teeth	Z, Z2		12	60
5	Radial Coefficient of Profile Shift	X,,, X,,,		0.34462	0
6	Involute Function $\alpha_{_{\mathrm{M}^{\prime}}}$	inv α _w	$2 \tan \alpha_i \left(\frac{x_{ii} + x_{ii}}{z_i + z_i} \right) + inv\alpha_i$	0.0183886	
7	Radial Working Pressure Angle	α_{m}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	y	$\frac{z_1 + z_2}{2} \left(\frac{\cos \alpha_t}{\cos \alpha_{nt}} - 1 \right)$	0.33333	
9	Center Distance	a,	$(\frac{z_1+z_2}{2}+y)m_t$	109.0000	
10	Standard Pitch Diameter	d	zm _t	36.000	180.000
11	Base Diameter	d,	d cosα,	33.8289	169.1447
12	Working Pitch Diameter	d _n	d ₅ cosα _{et}	36.3333	181.6667
13	Addendum	h _a , h _{az}	$(0.8796 + y - x_c) m_t$ $(0.8796 + y - x_c) m_t$	3.639	2.605
14	Whole Depth	h	$[1.8849 + y - (x_{ii} + x_{i2})]m_i$	5.621	
15	Outside Diameter	d _a	d + 2h,	43.278	185.210
16	Root Diameter	d,	d _a - 2h	32.036	173.968

No.	Item	Symbol	Formula	Example	
140.				Gear	Rack
1	Normal Module	m_a		2.5	
2	Normal Pressure Angle	α_o		20	P
3	Helix Angle	β		10° 57' 49"	
4	Number of Teeth & Helical Hand	Z		20 (R)	- (L)
5	Normal Coefficient of Profile Shift	X _m		0	-
6	Pitch Line Height	H		=0	27.5
7	Radial Pressure Angle	α_{t}	$\tan^{-1}\left(\frac{\tan \alpha_n}{\cos \beta}\right)$	20.34160*	
8	Mounting Distance	a,	$\frac{zm_n}{2\cos\beta} + H + x_n m_n$	52.9	65
9	Pitch Diameter	d	zm _n cosβ	50.92956	
10	Base Diameter	d _p	dcosa,	47,75343	
11	Addendum	h,	$m_a(1 + x_a)$	2,500	2.500
12	Whole Depth	h	2.25m,	5.62	25
13	Outside Diameter	d _a	d + 2 h,	55.929	76-0
14	Root Diameter	d,	d 2 h	44.679	-

No.	Item	Symbol	Formula	Example	
140.				Gear	Rack
1	Radial Module	m,		2.	5
2	Radial Pressure Angle	α_i		20°	
3	Helix Angle	β		10" 57' 49"	
4	Number of Teeth & Helical Hand	z		20 (R)	- (L)
5	Radial Coefficient of Profile Shift	х,		0	-
6	Pitch Line Height	Н		133	27.5
7	Mounting Distance	ax	$\frac{zm_t}{2} + H + x_t m_t$	52.5	00
8	Pitch Diameter	đ	zm _t	50.000	
9	Base Diameter	d _b	dcosα,	46.98463	-
10	Addendum	h,	$m_i(1 + x_i)$	2.500	2.500
11	Whole Depth	h	2.25m,	5.625	
12	Outside Diameter	d _a	d + 2 h,	55.000	
13	Root Diameter	d,	d, -2 h	43.750	-

The formulas of a standard helical rack are similar to those of **Table 6-6** with only the normal coefficient of profile shift $x_n = 0$. gear pair, the radial modules may not be the same. However, To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, ι , for one rotation of the mating gear is the product of the radial pitch, Pt and number of

$$I = \frac{\pi m_n}{\cos \beta} Z = p_t Z \tag{6-13}$$

According to the equations of **Table 6-7**, let radial pitch $P_t =$ 8 mm and displacement ι = 160 mm. The radial pitch and the displacement could be modified into integers, if the helix angle were chosen property.

1, for one turn of the helical gear equals the integral multiple of radial pitch.

(6-14) $\iota = \pi \text{ zm}$ SECTION 7 SCREW GEAR OR CROSSED HÉLICAL GEAR **MESHES**

These helical gears are also known as spiral gears. They are true helical gears and only differ in their application for interconnecting skew shafts, such as in Figure 7-1. Screw gears can be designed to connect shafts at any angle, but in most applications the shafts are at right angles.

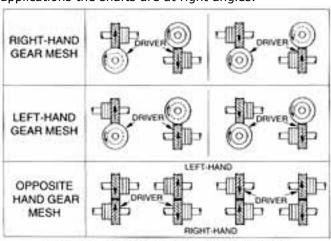


Fig. 7-1 Types of Helical Gear Meshes

NOTES:

- Helical gears of the same hand operate at right
- 2. Helical gears of opposite hand operate on parallel shafts.
- 3. Bearing location indicates the direction of thrust.

7.1 Features

7.1.1 Helix Angle and Hands

The helix angles need not be the same. However, their sum must equal the shaft angle:

$$\beta_1 + \dot{\beta}_2 = \Sigma \tag{7-1}$$

where β_1 and β_2 are the respective helix angles of the two gears, and Σ is the shaft angle (the acute angle between the two shafts when viewed in a direction paralleling a common perpendicular between the shafts).

Except for very small shaft angles, the helix hands are the same.

7.1.2 Module

Because of the possibility of different helix angles for the the normal modules must always be identical.

7.1.3 Center Distance

The pitch diameter of a crossed-helical gear is given by **Equation** (6-7), and the center distance becomes:

$$a = \frac{m_a}{2} \left(\frac{Z_1}{\cos \beta_1} + \frac{Z_2}{\cos \beta_2} \right)$$
 (7-2)

Again, it is possible to adjust the center distance by manipulating the helix angle. However, helix angles of both gears must be altered consistently in accordance with **Equation (7-1)**.

7.1.4 Velocity Ratio

Unlike spur and parallel shaft helical meshes, the velocity In the axial system, the linear displacement of the helical rack, ratio (gear ratio) cannot be determined from the ratio of pitch 'diameters, since these can be altered by juggling of helix angles. The speed ratio can be determined only from the number of teeth, as follows:

velocity ratio =
$$i = \underline{z}_1$$
 (7-3)

or, if pitch diameters are introduced, the relationship is:

i =
$$z_1 \cos \beta_2$$
 (7-4)
 $z_2 \cos \beta_1$

7.2 Screw Gear Calculations

Two screw gears can only mesh together under the conditions that normal modules, mn1 and, mn2 and normal pressure angles, m_{n1} m_{n2} , are the same. Let a pair of screw gears have the shaft angle ∞ and helical angles β_1 and β_2 :

If they have the same hands, then:
$$\Sigma = \beta_1 + \beta_2 \tag{7-5}$$
 If they have the opposite hands, then:

 $\Sigma = \beta_1 - \beta_2$, or $\Sigma = \beta_2 - \beta_1$ If the screw gears were profile shifted, the meshing would become a little more complex. Let β_{W1} , β_{W2} represent the

working pitch cylinder; If they have the same hands, then: $\Sigma = \beta_{wt} + \beta_{w2}$ (7-6)

If they have the opposite hands, then: $\Sigma = \beta_{w1} - \beta_{w2}$, or $\Sigma = \beta_{w2} - \beta_{w1}$

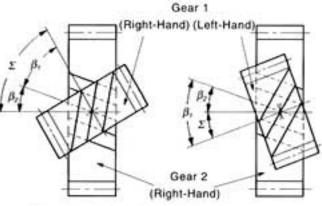


Fig. 7-2 Screw Gears of Nonparallel and Nonintersecting Axes